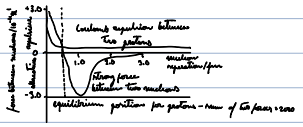
- Nuclear Physics
  - Formulas
    - Relationship between radius of nucleus and nucleon number: $R = R_0 A^{\frac{1}{3}}$
    - Decay equation for number of nuclei at time t: $N = N_0 e^{-\lambda t}$
    - Decay equation for activity at time t: $A = \lambda N_0 e^{-\lambda t}$
    - Angle of electron diffraction first min: $\sin\theta \approx \frac{\lambda}{D}$
  - Rutherford scattering and the nuclear radius
    - Main reasons of alpha scattering:
      - Most of α particles are undeflected
      - Some alpha particles deflected through wide angles
      - Some angles bounced back in opposite direction
    - Interpretations of these results were:
      - Most of atom is empty space
      - Atom contains small dense regions of electric charge
        - These small regions are positively charged.
    - Alpha particles which are sent back (backscatter) are alpha particles which have a head on collision with the gold nucleus.
      - Only $\frac{1}{8000}$ have a head on collision.
    - As alpha particles move closer to the nucleus its kinetic energy falls and its electrical potential energy increases.
      - When the alpha particle is at its closest to the nucleus, its kinetic energy has fallen to zero, and momentarily stopped moving.
    - Kinetic energy of particle and the electrical potential energy are related by: $E_\alpha = \frac{kZe \cdot 2e}{r_c}$
      - $k$ = proton number, $Z$ is proton number of gold (making $Ze$ the charge on gold nucleus), and $2e$ is alpha particle charge.
    - For alpha particles: $r_c = \frac{k2Ze^2}{E_\alpha}$.
      - In Rutherford's experiment $E_\alpha = 7.68$ MeV & $Z = 79$ meaning $r_c$ is $2.96 \cdot 10^{-14}$ m.
    - As the volume V of a nucleus must be proportional to the number of its nucleons, $V \propto A$ ($A$ is the nucleon number) and nuclear radius $R \propto A^{\frac{1}{3}}$. Giving: $R = R_0 A^{\frac{1}{3}}$.
      - $R_0$ is Fermi radius with value $1.2 \cdot 10^{-15}$ m.
    - Nuclear density
      - If the nucleus is spherical its volume would be: $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi A R_0^3$.
      - Density of nuclear material will be given by: $\rho = \frac{m}{V} = \frac{Au}{\frac{4}{3}\pi A R_0^3} = \frac{3u}{4\pi R_0^3} = \frac{3(1.66 \cdot 10^{-27})}{4\pi(1.2 \cdot 10^{-15})^3} = 2.3 \cdot 10^{17}\ \text{kg m}^{-3}$.
        - $u$ is the uniform atomic mass and $Au$ is the total mass of a nucleus of nucleon number $A$.
        - Density is independent of the number of nucleons in the nucleus.
        - The density of a single unified atomic mass is exceptionally dense, only a neutron star (stellar remnant resulting from gravitational collapse of a massive star), which is made of neutrons.
    - Deviations from Rutherford scattering
      - Having more energetic alpha particles will mean that the strong force will overcome the electrostatic repulsions.
      - Graphs



    - Electron Diffraction
      - Electrons are leptons (not hadrons) meaning they're affected by the charge distribution & not the strong force.
      - High energy electrons have short Broglie wavelengths of order $10^{-15}$ m.
      - For light incident on a small circular object of diameter D, the angle θ that the first diffraction minimum makes with the straight-through position ($\theta = 0°$) is given by
        - $\sin\theta \approx \frac{\lambda}{D}$
      - The intensity of the diffracted beam is seen to be a max in the straight-through position, falling to a min before slightly increasing again.
        - The minimum differs from light because it never reaches zero for electrons.
        - Given by formula: $\sin\theta = \frac{\lambda}{D}$, where $D$ is nuclear diameter & $\lambda$ is wavelength of electrons.
      - Required energy is supposed to be 400 MeV with wavelength given by: $\lambda = \frac{hc}{E} = \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{400 \cdot 10^{6} \cdot 1.6 \cdot 10^{-19}}$
        $= 3.1 \cdot 10^{-15}$ m
    - Worked Example

- $R = (1.2 \cdot 10^{-15})(40)^{\frac{1}{3}}$

  $R = \underbrace{4.10\ fm}_{\text{Radius, diameter} \approx 8.2\ fm}$

- $\lambda = \frac{hc}{E}$

  $\lambda = \frac{(6.63 \cdot 10^{-34})(3 \cdot 10^{8})}{420 \cdot 10^{6} \cdot 1.6 \cdot 10^{-19}}$

  $\lambda = 2.9 \cdot 10^{-15}\ m$

- $\sin\theta = \frac{\lambda}{D}$

  $\theta = \sin^{-1}\left(\frac{2.9 \cdot 10^{-15}}{9.08 \cdot 10^{-15}}\right)$

  $\theta = 18.6°$

- Because they're not close enough to the speed of light (≈400 MeV). Meaning their wavelength is too long.

## - Using electrons of higher energies

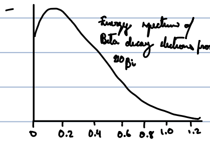
- When electrons of energy greater than 420 MeV are used for the scattering experiment, the collisions are no longer elastic (electrons lose $E_k$).
  - The energy is converted into mass as several mesons are emitted from the nucleus.
  - At even higher energies the electrons penetrate deeper into the nucleus & scatter off the quarks within protons & neutrons (deep inelastic scattering).

## - Energy levels in the nucleus

- Emission of gamma radiations is comparable to the emission of photons by electrons moving between different energy levels (orbitals).
- The emission of alpha or beta particles by radioactive nuclei often leaves the daughter nucleus in an excited state.
  - To go back to its ground state the daughter nucleus will emit one or more gamma ray photons.
- An example is americium-241, when it decays it'll emit an alpha particle, having one of a number of possible energies to become neptunium-237.
  - One of the alpha particles will bring the newly formed neptunium-237 to its ground state, while the others will leave it in an excited state.
    - If the nucleus is in an excited state it will emit one or two gamma photons to bring it to its ground state (number of gamma rays depends on which state it is in.
- The energy of the gamma photon can be calculated by seeing the difference between the energy levels & the ground state.
- $\alpha$ are quantized
- The $E_k$ of the nucleons in the atom is much lower than the amount required to escape the nucleus
  - Strong force provide $E_p$ barrier (when electrostatic repulsion will overcome strong force).
  - The higher the $E_p$ barrier the longer the half life.
- Quantum tunelling is where a subatomic particle passes through a $E_p$ barrier.

## - Negative Beta decay

- Anti-neutrino accompanies the electron in $\beta^-$ decay.
- Beta particles have a continuous energy spectrum and aren't discrete single energies like alpha particles & gamma photons.
  - $\beta^-$ energy spectrum



- The antineutrino was discovered as it would solve the mass-energy, momentum, and spin angular momentum to be conserved in emission.
  - Energy is shared between electron & antineutrino.

## - The law of radioactive decay

- Radioactive decay is a random & unpredictable process.
- The decay constant ($\lambda$) is the probability that an individual nucleus will decay in a given time interval (of an second/ hour/ year/ etc.)
  - Units are: $time^{-1}$ ($s^{-1}$, $min^{-1}$, $h^{-1}$, etc.).
- The activity of a sample $A$ is the number of nuclei decaying in a second (Bq).
- In sample of $N$ undecayed nuclei, the activity will be equal to the number of nuclei present times $\lambda^{-1}$.
  - $A = \lambda N$
  - $N$ will always decrease making it negative: $A = -\lambda N = \frac{\partial N}{\partial t} = -\lambda N$.
- $A = A_0 e^{-\lambda t}$ & $A = \lambda N_0 e^{-\lambda t}$.

## - Worked example

- $2200 = \lambda N$

  $N = \frac{2200}{1.35 \cdot 10^{-11}}$

  $N = 1.63 \cdot 10^{14}$

- $\frac{1.63 \cdot 10^{14}}{6.02 \cdot 10^{23}} = 2.7 \cdot 10^{-10}\ mol$

- $2.7 \cdot 10^{-10} \cdot 226 \cdot 10^{-3}\ kg$

  $= 6.1 \cdot 10^{-11}\ kg$

## - Worked example

- $10 \cdot 10^{-9}\ kg$

$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$

$= \frac{0.693}{2.1 \cdot 365 \cdot 24 \cdot 3600}$

$= \underline{1.05 \cdot 10^{-8}\ s^{-1}}$

$= 0.33 y^{-1}$

b) $10\ \mu g = 10 \cdot 10^{-9}\ kg$

$\frac{10 \cdot 10^{-9}}{134} = 7.5 \cdot 10^{-8}\ mol$

$7.5 \cdot 10^{-8} \cdot 6.02 \cdot 10^{23} = 4.49 \cdot 10^{16}$

$A = \lambda N$

$A = (1.05 \cdot 10^{-8})(4.49 \cdot 10^{16})$

$= 4.7 \cdot 10^{8}\ Bq$

c) $A = A_0 e^{-\lambda t}$

$A = (4.7 \cdot 10^{8}) e^{-(0.33)(10)}$

$A = 1.73 \cdot 10^{7}\ Bq$